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TESTING FOR STATIONARITY ON SELECTED LINEAR AND NON-LINEAR TIME SERIES MODELS OF DIFFERENT ORDERS

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Abstract

In this research, Understanding the assumptions of time series models is crucial. In this study, we aim to analyse and modify Autoregressive models on selected linear and nonlinear time series models to determine their stationarity and non-stationarity at different orders, it would be useful to conduct tests on the null hypothesis of a unit root. The study examines the power and type I error rates of different statistical tests, such as Kwiatkowski, Phillips, Schmidt and Shin (KPSS), Augmented Dickey-Fuller (ADF), and Phillips Perron (PP). These tests are used to determine whether a given dataset is stationary or non-stationary. The study also considers different orders of Autoregressive (AR) and Trigonometric Smooth Transition Autoregressive (TSTAR) models, as well as various sample sizes. A Python software was utilised to conduct an investigation simulating the performance of stationarity and unit root tests. The experiments were carried out using sample sizes; 20, 50, 70, 100, 120, 150, 180, 200, 230 and 250 for various orders of Autoregressive (AR) and Trigonometric Smooth Transition Autoregressive (TSTAR) models. The tests were compared based on their respective power percentages to determine their relative performance. The study determined that Phillips Perron (PP) is the optimal choice across all the conditions examined, including the models, sample sizes, and ordering.

Keywords: Stationarity, Autoregressive Models, Different Orders, Linear and Non-Linear Models

1. INTRODUCTION

A time series model is a collection of data points arranged in chronological order, with time serving as the independent variable. Also, Time series is an order in which data points are collected in a sequential manner over a period of time. A time series data is indeed an array of time and numbers. Data obtained from observations collected at successive time space are widely available. For example, there are monthly or yearly sales, quarterly demands and yearly price indices in economic studies. Also, yearly, monthly and daily precipitation, flooding, temperatures and wind speeds are observed in meteorology. It is equally observed that in agriculture, soil erosion, crop and livestock productions, and number of eggs produced by fowls are recorded daily, monthly and annually. The area of application of time series is virtually endless.

A time series model typically has the component for the mean and the component for the conditional variance (Box and Pierce, 1970). Time series is an order in which data points are collected in a sequential manner over a period of time. A time series data is indeed an array of time and numbers. Moving average, autoregressive moving average, exponential, autoregressive, and seasonal modeling proceed through a sequence of well-defined procedures. The initial stage involves identifying the model. Identification involves precisely determining the suitable configuration. (Autoregressive (AR) models, Smooth Transition Autoregressive, Exponential Smooth Autoregressive model and Trigonometric model) whether the model is stationary or non-stationary under distributions. Identification is occasionally performed by examining plots of the autocorrelation function and partial autocorrelation function. Identification is occasionally carried out by an automated iterative process that involves fitting numerous potential model structures and ordering. A goodness-of-fit statistic is then used to determine the most suitable model.

A. Stationarity

A stationary process in Statistics refers to a stochastic process that maintains the same joint probability distribution even when it is shifted in time. Therefore, if the mean and variance exist, they remain constant across time.

The primary assumption regarding time series data is that it is stationary. Stationarity refers to the concept that the probability principles governing the behaviour of a process remain constant throughout time. Yes, the process is in a state of statistical equilibrium. More precisely, a process $\{Y\_t\}$ is considered rigorously stable if the joint distribution of Yt is identical to that of Yt − k for all values of t and k, where t and k are positive integers. Put simply, the Y's have a nearly same distribution (Jonathan and Kung-Sik, 2008). Consequently, it can be deduced that the expected value of Yt is equal to the expected value of Yt - k for all values of t and k, resulting in a constant mean function at all points in time. Similarly, the variance of Yt is equal to the variance of Yt - k for all values of t and k, indicating that the variance remains constant across time. Furthermore, a fundamental premise of stationary time series is the presence of white noise. This means that the error term in the model must follow a normal distribution with a mean of zero and a variance of σ2.

B. Nonlinear Time series Model

Time series data possessing nonlinear attributes are increasingly faced by practitioners in many fields. Stationary Gaussian autoregressive models are determined by their first two moments. Therefore, it is necessary for linear autoregressive models to exhibit time reversibility. It is worth noting that several real datasets exhibit time irreversibility, which implies that the underlying process is nonlinear (David and Darfiana, 2011). As discussed in the influential paper by Tong and Lim (1980), it was demonstrated that the cyclical dynamics observed in sections of the lynx data cannot be adequately explained by a linear Gaussian model. In addition, he made a compelling case for the necessity of practical nonlinear models in addressing the persistent challenges posed by characteristics like

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time irreversibility and limit cycles in real data. The explanation and application of locally linear threshold models by Tong (1990) opened up exciting possibilities for model building strategies.

The identification, modeling and forecasting time series data that have nonlinear attributes have attracted considerable attention in different fields of studies such as engineering, social science, life science, natural sciences, financial studies and other related areas which exhibit nonlinear process. When a nonlinear attribute is determined in data, strong evidence has been found in the data to abandon the linear model and therefore a nonlinear model needs to be selected. In order to select a suitable nonlinear model for time series data, good statistical and diagnostic tests are needed to determine the nonlinearity in the data.

2. LITERATURE REVIEW

Next, we need to calculate the coefficients of the model. Estimating coefficients of AR models can be achieved through the method of least-squares regression. Understanding and estimating parameters of MA and ARMA models typically involves a more intricate iteration process (Chen and Tsay, 2019). In practice, estimation is easily handled by a computer programme, requiring minimal user interaction. This ensures a transparent process for the user. Next, we need to verify the model. This step is also known as diagnostic checking, or verification (Anderson, 1976). Ensuring that the residuals of the model are random and that the estimated parameters are statistically significant are two crucial aspects of the checking process. Typically, the fitting process is driven by the principle of simplicity. The goal is to find the most straightforward model with the fewest parameters that accurately represents the data.

A discrete time series consists of a collection of time-ordered values $\{y_1, y_2 \dots \dots \dots \dots \dots y_n\}$ derived from observations of a particular phenomenon., A time series is a sequence of observations that are measured sequentially throughout time. These measurements can either be continuous or performed at specific time intervals (Kim, 2022). The key techniques for handling econometrics and time series data in model fitting are Autoregressive (AR) models, Smooth Transition Autoregressive models, Exponential Smooth Autoregressive models, and Trigonometric models. The fundamental premise of these models is the need of stationarity, meaning that the data being used for fitting should exhibit stationarity.

Autoregressive Mathematical models such as smooth transition and moving average models capture the persistence, or autocorrelation, in a time series. The models are commonly utilised in econometrics, hydrology, engineering, and various other fields. There are several reasons why fitting Autoregressive (AR) models, Smooth Transition Autoregressive (STAR), Exponential Smooth Autoregressive model, and Trigonometric model models to data is beneficial. Modelling can provide valuable insights into the physical system by shedding light on the underlying processes that contribute to the persistence observed in the series. These models can also be utilised to forecast the behaviour of a time series or econometric data based on historical values. This prediction can serve as a reference point for assessing the potential significance of other variables

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in the system. They have become a common tool for forecasting economic and industrial time series. Autoregressive (AR) models, Smooth Transition Autoregressive (STAR), Exponential Smooth Autoregressive model, and Trigonometric model can also be used for simulation purposes. These models allow for the generation of synthetic series that mimic the persistence structure of an observed series. Simulations are particularly valuable for establishing confidence intervals for statistics and estimated econometrics quantities.

Based on these studies, it would be beneficial to conduct tests for both stationarity and a unit root when determining the nature of economic data using classical methods. This paper presents a clear and concise examination of the null hypothesis of stationarity compared to the alternative of a unit root. It explores different orders of autoregressive and moving average, as well as various sample sizes. It is quite remarkable that there have been very few previous attempts to test the null hypothesis of stationarity. In their study, Park and Mahdi (2020) examine a test statistic that resembles the F statistic for deterministic trend variables that are considered unnecessary. According to their findings, this statistic should ideally be close to zero when the null hypothesis of stationarity holds true, but not when there is an alternative hypothesis of a unit root. In their study, Zhang and Zhou (2022) examine the Dickey-Fuller test statistics by estimating both trend-stationary and difference-stationary models. They further employ the bootstrap method to assess the distribution of these statistics.

Autoregressive, exponential, autoregressive moving average, moving average, and seasonal modeling Follow a structured sequence of clear steps. Identifying the model is the initial step. Identification involves determining the suitable structure (Autoregressive (AR) models Smooth Transition Autoregressive, Exponential Smooth Autoregressive model and Trigonometric model whether the model is stationary or non-stationary under distributions. Identification can be performed by examining plots of the autocorrelation function and partial autocorrelation function. Identification is often achieved through an automated iterative process that involves fitting various model structures and orders. A goodness-of-fit statistic is then used to determine the most suitable model.

Next, we need to determine the coefficients of the model. The coefficients of AR models can be estimated using least-squares regression. Estimating parameters of Autoregressive models typically involves a more intricate iteration process (Kim, 2022). In practice, the process of estimation is quite straightforward for the user, as it is carried out automatically by a computer programme with minimal or no user involvement. Next, we need to examine the model. This step is also known as diagnostic checking, or verification (Anderson, 1976). It is crucial to verify that the residuals of the model exhibit randomness and that the estimated parameters demonstrate statistical significance. Typically, the fitting process is driven by the idea of simplicity. The goal is to find the most straightforward model with the fewest parameters that can accurately represent the data.

A stationary process in Statistics refers to a stochastic process that maintains a consistent joint probability distribution even when shifted in time. Therefore, if there are any parameters like the mean and variance, they remain constant over time. Stationarity is a crucial assumption when dealing with time series data. Stationarity is a fundamental concept in understanding how the behaviour of a process remains consistent over time. It revolves around the notion that the probability laws governing the process do not

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undergo any changes as time progresses. Truly, the process is in a state of statistical equilibrium. In order to determine if a process {Y_t} is strictly stationary, we need to compare the joint distribution of Yt with that of Yt − k for all t and k, where t = 1, 2, … k. As stated by Jonathan and Kung-Sik (2008), the Y's are (marginally) identically distributed. It can be deduced that the expected value of Yt is equal to the expected value of Yt - k for all values of t and k, indicating that the mean function remains constant over time. The variance remains constant over time because Var (Yt) = Var(Yt − k) for all t and k. Furthermore, it is essential to note that in the realm of stationary time series, a fundamental assumption is the presence of white noise. This implies that the error term of the model must adhere to a normal distribution, with a mean of zero and a variance of σ^2.

The classical model for the mean part is the autoregressive (AR) model. When analysing time series data, it is common to discuss the concept of stationarity, which refers to the behaviour of a specific variable over time. There are three components to stationarity. The series exhibits a constant mean, indicating that there is no inherent inclination for the mean of the series to either rise or fall over time. Additionally, it is assumed that the variance of the series remains constant over time. Finally, it is assumed that the autocorrelation pattern remains constant throughout the series. Over the past twenty years, numerous non-linear time series models have been put forward. These include the bilinear model by (Granger and Andersen, 1978), the amplitude dependent exponential AR (EX PAR) model by Maulana and Slamet, (2020), the threshold AR model by Fan, (2019), and the random coefficient of AR model by (Ratnasingam and Ning, 2020), among several others.

An influential model for the conditional variance component is the autoregressive conditional heteroscedastic (ARCH) model, as proposed by Kim et al. (2022). various time series models are commonly employed to fit and analyses the dynamic behavior of time series data. The commonly used ones are Linear models, including autoregressive (AR) models, moving average (MA) models, and mixed autoregressive moving average (ARMA) models (Chung-Ming, 2002). The ARCH model provides a fresh tool for quantifying risk and its influence on investment returns. It also offers a fresh approach to pricing and hedging non-linear assets like options. In 2023, Bollerslev introduced the GARCH model, which is a generalization of the ARCH model. Hipel and McLeod, (1994) hypothesized that, although a linear model may be adequate to describe average annual river flows, the relationship between daily river flow and precipitation may be nonlinear. For examples, Sankar & Pushpa, (2022) provides an introduction to different types of nonlinear time series modeling primarily in the univariate setting. Franses and van Dijk, (2020) investigated the techniques for obtaining bivariate nonlinear models. Terasvirta, (2022) mentioned vector nonlinear autoregressive processes, vector nonlinear average processes and multiple bilinear time series models in passing but concentrated on statistical inference for nonlinear models using parametric procedure.

The linear time series modeling depends on the type of system that generates the data. According to Kung-Sik (2018) and Liu and Zhu, (2022), time series analysis can be done using autoregressive (AR), moving average (MA), or autoregressive moving average (ARMA) models. Time series data possessing nonlinear attributes are increasingly faced by practitioners in many fields. It is well-known that the first two moments structurally

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determine stationary Gaussian autoregressive models. Therefore, linear autoregressive models must be time reversible. However, many real datasets are time irreversible, indicating a nonlinear process (David and Darfiana, 2011). In fact, in the seminal paper on threshold models by Tong and Tsay, (2022), it was argued that no linear Gaussian model could account for the cyclical dynamics seen in sections of the lynx data. Tong and Lim, (2019) demonstrated that the threshold model can produce asymmetrical periodic behavior, as seen in the annual Wolf's sunspot and Canadian lynx data.

So far, it has been suggested to use autoregressive models. As an illustration, consider the models presented by Nelson, (2019) for exponential functions, Alok, (2020) for absolute value functions, and Soltyk and Chan, (2023) for non-linear functions. It is challenging to ascertain the functional forms of the mean and conditional variance components in these models. A crucial part of any statistical model is, therefore, to verify these functional forms. Box and Pierce (1970) introduced a portmanteau test that utilizes the residual autocorrelation function (ACF) to assess the adequacy of a model. The test was revised by Franse and van Dijk, (2020) to enhance its effectiveness. In their study, Fathian and Nadoushani, (2019) expanded their method to encompass a broader range of scenarios. In their recent study, Mcleod and Li, (2021) introduced a novel portmanteau test that utilizes squared residuals to analyze the ARMA model. These tests specifically address ARMA models and are not intended for non-linear time series models. In a study conducted by Li, (1992), the focus was on examining the asymptotic standard errors of the residual ACF in non-linear time series models. In their study, Cheng and Gan, (2020) proposed a statistical method that builds upon the Box-Pierce statistic. This method incorporates the first M-squared standardized residual ACF to assess various non-linear ARCH specifications. In their study, Li and Mak, (2019) introduced a comprehensive class of squared residual ACF to assess the validity of non-linear time series models, such as ARMA, ARCH, and other similar models.

3. MATERIALS AND METHODS

3.1 Data source and specifications

In this section, we will conduct simulation studies to examine the performance of tests for stationarity across various orders of autoregressive and trigonometric models. $Y_t = \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \ldots + \emptyset_p Y_{t-p} + e_t$, where e_t represents a white noise process with a mean of zero and a variance of σ^2. $\emptyset_1, \emptyset_2, \ldots \emptyset_p$ are autoregressive parameters. The mathematical model for existing and proposed models are $Y_t = \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \ldots + \emptyset_p Y_{t-p} + e_t$ is an existing model for Autoregressive model, where e_t represents a white noise process with a mean of zero and a variance of σ^2.

$Y_t = TSTAR_{(p)}$ is $= Sin\phi_1 Y_{t-1} + Sin\phi_2 Y_{t-2} + \cdots + Sin\phi_p Y_{t-p} + e_t$ is called Trigonometric Smooth Autoregressive (TSTAR) model.

3.2 Test of Stationarity and Unit Root (non-Stationarity)

Let's take a look at the statistical issues that arise when dealing with autoregressive unit root tests, using the example of a simple $AR_{(p)}$ model.

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$$Y_t = \emptyset_1 Y_{t-1} + e_t, e_t \sim WN(0, \sigma^2) \quad (1)$$

H_0: $\emptyset = 1\ vs H_1: |\emptyset| < 1$

Here is the test statistic:

$$t_{\emptyset=1} = \frac{\widehat{\emptyset} - 1}{SE(\widehat{\emptyset})} \quad (2)$$

Given that $\widehat{\emptyset}$ represents the least square estimate and SE ($\widehat{\emptyset}$) is the usual standard error estimate, it can be concluded that the test is a one-sided left tail test. If Y_t is stationary (i.e., if the absolute value of $\emptyset$ is less than 1), it can be demonstrated by Hamilton (1994) that; $t_{\emptyset=1} \sim N(0,1)$

3.3 Augmented Dickey–Fuller test

The augmented Dickey-Fuller test (ADF) is a statistical test used to determine if a time series sample has a unit root. This test is an enhanced version of the Dickey-Fuller test, designed to handle larger and more complex time series models. In the test, the ADF statistic is a negative number. If the results are more negative, it would indicate a stronger rejection of the hypothesis that there is a unit root at a certain level of confidence. Next, the unit root test is conducted, comparing the null hypothesis $\emptyset = 0$ with the alternative hypothesis of $\emptyset < 0$. After determining a value for the test statistic

$$DF = \frac{\widehat{\emptyset}}{SE(\widehat{\emptyset})} \quad (3)$$

is calculated, it can be compared to the appropriate critical value for the Dickey-Fuller Test. If the test statistic is smaller (since this test is non-symmetrical, we do not consider an absolute value) than the critical value (which is more negative), then we reject the null hypothesis of$\emptyset = 0$ and conclude that no unit root is present.

3.4 Kwiatkowski, Phillips, Schmidt and Shin (KPSS)

In their study, Kwiatkowski *et al.* (2016) introduced the Langrage Multiplier (LM) test as a means of assessing trend and/or level stationarity, also known as the KPSS test. Now, the null hypothesis assumes a stationary process. Considering the null hypothesis as a stationary process and the unit root as an alternative is in line with a cautious testing

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approach. Therefore, if we reject the null hypothesis, we can have a high level of confidence that the series does have a unit root. If the tests show a unit root but the KPSS test shows a stationary process, it is advisable to proceed with caution and consider the latter result.

3.5 Null hypothesis

$$H_0: \sigma_e^2 = 0 \quad (4)$$

Assuming that the errors follow a normal distribution with mean zero and variance σ_e^2 ($e_t \sim NIID(0, \sigma_e^2)$), the test statistic can be calculated.

$$LM = \frac{\sum_{t=1}^{T} S_t^2}{\hat{\sigma}_e^2} \quad (5)$$

$$\hat{\sigma}_e^2 = \frac{\sum_{t=1}^{T} e_t^2}{T} \quad (6)$$

$$S_t = \sum_{i=1}^{t} e_i, t = 1, \ldots, T \quad (7)$$

The temporal trend and constant Y_t on are used in the regression, and the residuals, denoted as e_t , are involved.

3.6 Phillips Perron Test

The Phillips Perron test evaluates the hypothesis of a unit root in modeling time series Y_t. The test equation, PP = $c + \sigma_t + \phi_1 Y_{ti-1} + e_t$, includes the drift coefficient c and the deterministic trend coefficient σ.

4. RESULTS AND DISCUSSION

In this section, numerical simulation study was conducted for the Autoregressive and Trigonometric Smooth Autoregressive models at different order under stationary and to investigate the Effect of sample size of each model at different orders were examined on each of the models. Tables 1 and 2 present the accuracy of identifying stationarity in the autoregressive (AR) models and stationarity in the Trigonometric Smooth Autoregressive (TSTAR) models respectively. Similarly, figures 1 and 2 present the stationarity Test Power for Autoregressive Models and Stationarity Test Power for TSTAR Models of Various Orders respectively.

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Table 1: Accuracy of identifying stationarity in the autoregressive (AR) models expressed as a percentage.

H₀: $|\phi_j| = 1(nonstationary)\ vs H_1: |\phi_j| < 1(stationary)$. Models: AR(1) = $Y_{ti} = 0.5Y_{t-1} + e_t$, AR(2): $Y_{ti} = 0.5Y_{t-1} + 0.3Y_{t-2} + e_t$,AR(3): $Y_{ti} = 0.5Y_{t-1} + 0.3Y_{t-2} + 0.1Y_{t-3} + e_t$

Test Statistic	ADF			KPSS			PP		
Order	1	2	3	1	2	3	1	2	3
Sample Size(n)									
20	80.15	79.00	75.87	70.03	70.35	70.04	94.52	94.52	97.00
50	80.24	79.60	77.78	72.53	72.85	72.54	94.55	95.54	97.00
70	82.45	79.90	77.80	74.53	74.85	74.54	94.59	96.52	97.00
100	82.25	79.99	80.08	74.73	75.05	74.74	94.95	96.50	97.00
120	83.55	79.99	80.07	71.03	75.35	75.04	95.58	96.51	97.00
150	85.25	85.25	85.25	74.00	77.85	77.05	97.00	97.00	97.00
180	85.25	85.25	85.25	75.45	77.85	77.11	97.00	97.00	97.00
200	85.25	85.25	85.25	75.47	77.85	77.21	97.00	97.00	97.00
230	85.25	85.25	85.25	76.45	77.85	77.45	97.00	97.00	97.00
250	85.25	85.25	85.25	77.53	77.85	77.54	97.00	97.00	97.00

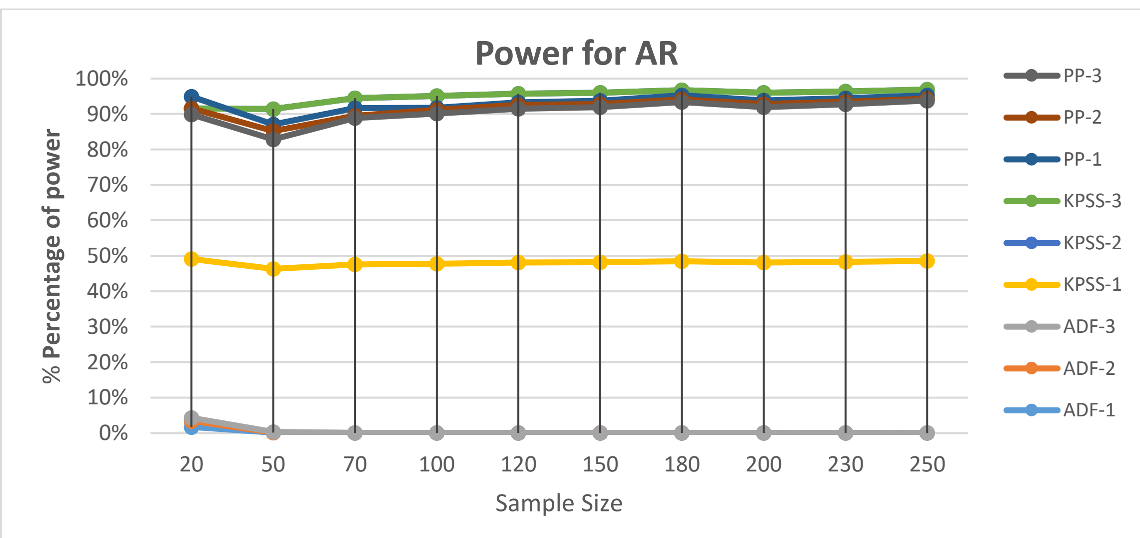

Figure 1: Stationarity Test Power for Autoregressive Models of Various Orders

Table 2: Accuracy in identifying stationarity in the Trigonometric Smooth Autoregressive (TSTAR) models expressed as a percentage.

H₀:$|\phi_j| = 1(nonstatinary)\ vs H_1: |\phi_j| < 1(stationary)$. Models: TSTAR(1): $Y_{ti} = sin0.5Y_{t-1} + e_t$, TSTAR(2): $Y_{ti} = sin0.5Y_{t-1} + sin0.3Y_{t-2} + e_t$, TSTAR(3): $Y_{ti} = sin0.5Y_{t-1} + sin0.3Y_{t-2} + sin0.1Y_{t-3} + e_t$.

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Test Statistic			**ADF**	**KPSS**				**PP**	
Order	1	2	3	1	2	3	1	2	3
Sample Size(n)									
20	55.87	55.76	45.72	32.98	31.54	30.55	90.12	90.12	90.12
50	55.87	55.76	45.72	32.98	31.54	30.55	90.34	90.34	90.34
70	55.87	55.76	45.72	32.98	31.54	30.55	90.45	90.45	90.45
100	55.87	55.76	45.72	32.98	31.54	30.55	90.45	90.45	90.45
120	55.87	55.76	45.72	32.98	31.54	30.55	90.75	90.75	90.75
150	55.87	55.76	45.72	32.98	31.54	30.55	91.45	91.45	91.45
180	55.87	55.76	45.72	32.98	31.54	30.55	91.45	91.45	91.45
200	55.87	55.76	45.72	32.98	31.54	30.55	91.45	91.45	91.45
230	55.87	55.76	45.72	32.98	31.54	30.55	91.45	91.45	91.45
250	55.87	55.76	45.72	32.98	31.54	30.55	92.23	92.23	92.23

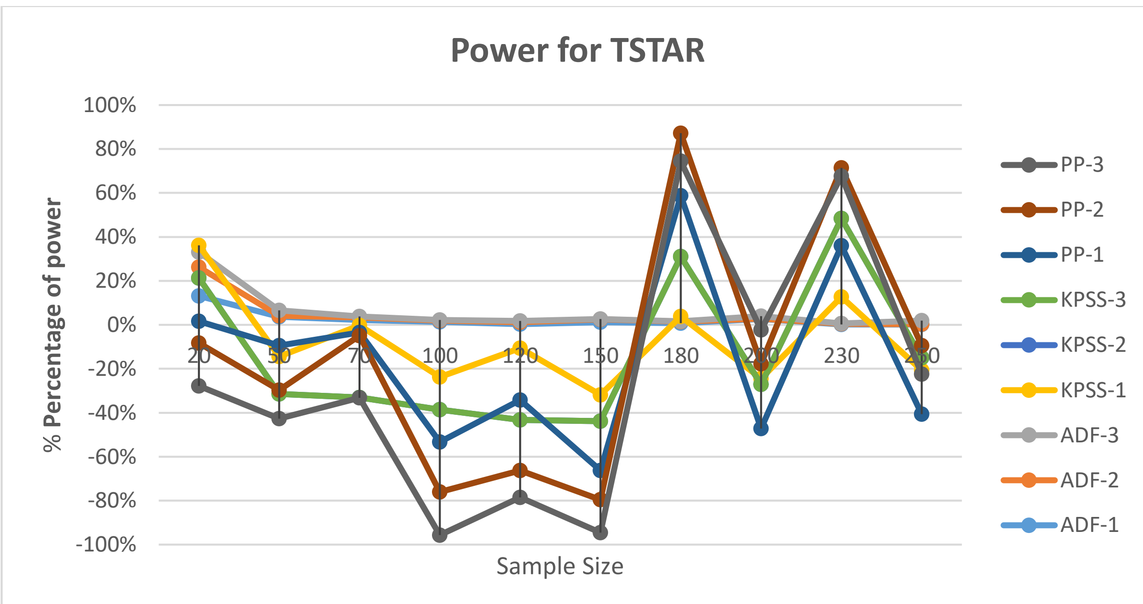

Figure 2: Stationarity Test Power for TSTAR Models of Various Orders

5. CONCLUSION

In this study, we tested for the stationarity of the modified Autoregressive (AR) and relative performance orders of Trigonometric Smooth Transition Autoregressive (TSTAR) models. The power and kind. The errors of each test were analyzed by doing simulations at various sample sizes and for different orders (p) of the models. The acceptance rates for each order of the models, including Autoregressive (AR) and Trigonometric Smooth Transition Autoregressive (TSTAR) models, as well as the sample sizes, were recorded in a table. The results obtained were plotted on the graphs as shown in figure 1 and 2. The test that had a greater acceptance rate was regarded as the most effective approach. The differences in the results of three stationarity and unit root tests will indicate how sensitive the

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approaches are. Therefore, the Phillips Perron method was chosen as the most suitable approach for this investigation, considering several sample sizes and models such as Autoregressive (AR) and Trigonometric Smooth Transition Autoregressive (TSTAR) models.

References

Abdel Zaher, M., Elbarbary, S., Sultan, S.A., El-Qady, G., Ismail, A., & Takla, E.M. (2018a). Crustal thermal structure of the Farafra oasis, Egypt, based on airborne potential field data. Geothermics 75, pages 220–234. https://doi.org/10.1016/j.geothermics.2018.05.006

Amelot, L. M. M., SubadarAgathee, U., & Sunecher, Y. (2021). Time series modelling, NARX neural network and hybrid KPCA–SVR approach to forecast the foreign exchange market in Mauritius. *African Journal of Economic and Management Studies*, *12*(1), 18-54.

Barnett, D.F. Hendry, S. Hylleberg, T. Ter¨asvirta, D. Tjøstheim and A.H. W¨urtz (eds.), *Nonlinear Econometric Modeling*, Cambridge: Cambridge University Press.

Barrientos, A. F., &Canale, A. (2021). A Bayesian goodness-of-fit test for regression. *Computational Statistics & Data Analysis*, *155*, 107104.

Boero, G. and Marrocu, E. (2018). The performance of SETAR models: a regime conditional evaluation of point, interval and density forecasts, Working Paper.

Boero, G. and Marrocu, E. (2022). The performance of non-linear exchange rate models:a forecasting comparison, Journal of Forecasting, Vol. 21(7), 513-542.

Bollerslev, T. (2023). Reprint of: Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*.

Box, G. E. P., Jenkins, G. M., and Reinsel, G. C. (1994). *Time Series Analysis, Forecasting and Control,* 2nd ed. New York: Prentice-Hall.

Cheng, C., Guan, Y., Chen, G., &Gan, M. (2020,). Parameters Estimation of RBF-AR Model Based on EM and Variable Projection Algorithm. In *2020 Chinese Control And Decision Conference (CCDC)* (pp. 1678-1683). IEEE.

CHIADMI, M. (2020). ISLAMIC STOCK MARKET: Modeling Volatility and Comparative Study. *International Journal of Economics and Financial Issues*, *1*(1-2), 11-

Choi I (2006). Nonstationary Panels." In TC Mills, K Patterson (eds.), Econometric Theory, volume 1 of Palgrave Handbook of Econometrics, chapter 13, pp. 511{539. Palgrave MacMillan, New York

Clements, M. and Smith, J. (2001). Evaluating forecasts from SETAR models of exchange rates, Journal of International Money and Finance, Vol. 20(1),133-148.

Clements, M., and Smith, J. (2018): A Monte Carlo Study of the Forecasting Performance of Empirical SETAR Models. Journal of Applied Econometrics 14: 123–41.

Corbet, S., Hou, Y. G., Hu, Y., & Oxley, L. (2021). An analysis of investor behavior and information flows surrounding the negative WTI oil price futures event. *Energy Economics*, *104*, 105589.

Dickey, D.A. and W.A. Fuller (2009). Distribution of the estimators for autoregressive time series with a unit root, Journal of the American Statistical Association 74, 427-431.

Dickey,D. and W.fuller (1981).Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root" Econometrica, 49,1057-1072.

Dickey,D. and W.fuller.(1979).Distribution of the Estimations for Autoregressive Time Series with a Unit Root" Journal of the American Statistical Association, 74,427-431.

Diebold, F. X. and J. A. Nason, (1990). Nonparametric exchange rate prediction, Journal of

Ding, S., Shi, X., & Wu, Y. (2020). Beta approximation and its applications. *Journal of Statistical Computation and Simulation*, *90*(7), 1251-1266.

Enders, W.(2009). *Applied Econometric Time Series*: John Wiley & Sons.

Franses , P.H. and van Dijk, D. (2020). Nonlinear error-correction models for interest rates in the Netherlands.

Fuller,W.(1996).Introduction to Statistical Time Series ,Second Edition .john willey, New York

Gairola, G., &Dey, K. (2022). Price discovery and risk management in asset class: a bibliometric analysis and research agenda. *Applied Economics Letters*, 1-12.

Granger, C., and Teräsvirta, T. (2018). Modeling Nonlinear Economic Relationships. Oxford University Press, New York.

Hamilton J. D. (1994). *Time Series Analysis*. Princeton University Press, Princeton, NJ.

Hargreaves, C. P. (2019). *Nonstationary time series analysis and cointegration*: Oxford

Hargreaves, Colin P. (1994). *Nonstationary time series analysis and cointegration*: OxfordUniversity Press.

Harvill, J. L. and Ray, B.K (1999). A note on tests for nonlinearity in a vector time series, Biometrika, 86(3), 728–734.

Hipel, K.W. and McLeod, A.I. (1994). Time series Modeling of Water Resources and Environmental Systems. Amsterdam: Elsevier.

Maulana, A., &Slamet, I. (2020,). Prediction of Indonesian financial crisis using Markov regime switching autoregressive conditional heteroscedasticity models based on bank deposits and lending/deposit interest rate indicators. In *Journal of Physics: Conference Series* (Vol. 1563, No. 1, p. 012017). IOP Publishing.

Mcleod, A.I. and Li, W.K. (2019) Diagnostic checking ARMA time series models using squared-residual autocorrelation. Journal of Time Series Analysis 4, 269-273.

Okorie, D. I., & Lin, B. (2021). Stock markets and the COVID-19 fractal contagion effects. *Finance Research Letters*, *38*, 101640.

Oulhiq, R., Benjelloun, K., Kali, Y., &Saad, M. (2022). A data mining based approach for process identification using historical data. *International Journal of Modelling and Simulation*, *42*(2), 335-349.

Salcedo-Sanz, S., Casillas-Pérez, D., Del Ser, J., Casanova-Mateo, C., Cuadra, L., Piles, M., & Camps-Valls, G. (2022). Persistence in complex systems. *Physics Reports*, *957*, 1-73.

Sankar, T. J., &Pushpa, P. (2022). Implementation of Time Series Stochastic Modeling for Zea mays Production in India. *Mathematical Statistician and Engineering Applications*, *71*(4), 611-621.

Terasvirta, T., (2020). The Performance of Ramsey test and white test of Terasvirta Test in detecting Nonlinearity.

Terasvirta, T., Tstheim, D., & Granger, C. W. (2022). Modelling nonlinear economic time series. University Press.

Wątorek, M., Drożdż, S., Kwapień, J., Minati, L., Oświęcimka, P., &Stanuszek, M. (2021). Multiscale characteristics of the emerging global cryptocurrency market. *Physics Reports*, *901*, 1-82.

Yang, Y., Ling, S., & Wang, Q. (2022). Consistency of global LSE for MA (1) models. *Statistics & Probability Letters*, *182*, 109292.

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Zellner A, Palm FC (1974). \Time Series Analysis and Simultaneous Equation Econometric Models." Journal of Econometrics, 2(1), 17- 54

Zhang, J. T., Guo, J., & Zhou, B. (2022). Testing equality of several distributions in separable metric spaces: A maximum mean discrepancy-based approach. *Journal of Econometrics*.

Zhang, J., Li, B., Liu, X., & Wan, X. (2023). Asymptotic behavior of the portmanteau tests in an integer-valued AR model. *Journal of Nonparametric Statistics*, 1-25**.**

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